The Structural Origin of Schrödinger-Type Dynamics from JS–SH Discrete Geometry

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Abstract

We analyze, from first principles, the minimal structural conditions under which Schrödinger-type evolution equations appear as effective continuum descriptions of conservative wave-like systems.

Our starting point is a discrete structural framework composed of JS-cells and SH-hub couplings, but the logical implication is *not* reversed: we do not claim that all Schrödinger dynamics originates from JS–SH structures.

No quantum-mechanical postulates, complex wavefunctions, canonical quantization rules, or predefined physical constants are assumed at the outset. Instead, we identify minimal structural conditions under which a Schrödinger-type continuum description emerges as a *structure-hiding effective representation* of a more primitive real two-channel conservative law.

The analysis proceeds from three elementary requirements: (i) a real two-channel internal state, (ii) local structural connectivity, and (iii) strict conservation of a quadratic norm. We show that whenever these conditions are satisfied, an internal $SO(2)$ rotational sector is unavoidable, with generator matrix $\mathbf{J}$ playing the role of an emergent phase operator. In such cases, the resulting first-order Schrödinger-type equation does not constitute a fundamental dynamical principle, but rather a structure-hiding compression of an underlying real wave–rotation dynamics.

Crucially, we demonstrate that this hierarchy admits a sharp structural distinction between phase-closed and phase-open regimes. When the underlying wave process achieves structural phase closure (i.e. completed wavelength cycles with matched initial and terminal states), the dynamics reduces to stationary normal modes. Conversely, when phase closure is not achieved, the residual phase mismatch necessarily evolves according to a Schrödinger-type envelope equation. In this sense, Schrödinger evolution emerges as the generic envelope description, within this structural class, for the dynamics of *structurally incomplete wave processes*.

Within the JS–SH framework, these mechanisms are realized explicitly, allowing the Schrödinger equation to be identified as a *special continuum representation* (long-wavelength, phase-open limit) of a more general discrete wave–rotation master law, in direct analogy with how the classical wave equation emerges from discrete structural connectivity.

1 Introduction

The Schrödinger equation occupies a central position in modern physics, yet its conceptual status remains unusual.

Relation to companion CMP submissions. This manuscript complements our ongoing CMP submissions on (i) the discrete-to-continuum derivation of the wave equation, (ii) the structural origin of the gravitational constant, and (iii) the emergence of discreteness on ordered continua, by focusing specifically on Schrödinger-type dynamics as an effective continuum representation of a real two-channel conservative structure. No results are duplicated; the present work isolates the minimal rotational sector and the norm-preserving locality that force the Schrödinger universality class.

In standard presentations, it is postulated rather than explained: the complex wavefunction, the imaginary unit, and Planck's constant are introduced axiomatically.

In this work, we take a different perspective. We do not claim that the Schrödinger equation has a single universal microscopic origin. Instead, we address a more precise structural question:

> *What minimal internal structure must be present whenever a Schrödinger-type continuum description is valid?*

Our analysis shows that Schrödinger-type dynamics does not arise arbitrarily. Whenever such an equation provides a faithful effective description, the underlying system necessarily contains: (i) a real two-channel rotational sector, (ii) a positive-definite locally conserved scalar density, and (iii) a continuity-enforced flux structure. These ingredients together force the canonical Schrödinger form, independently of any quantum postulates.

To make this structure explicit, we introduce a concrete discrete realization built from two elements:

- JS-**cells**, carrying a two-component real internal state, and
- SH-**hubs**, mediating local linear connectivity.

The JS–SH framework is therefore *not* asserted as the unique origin of Schrödinger dynamics. Rather, it provides a minimal and explicit realization of the structural conditions that Schrödinger-type equations implicitly encode.

Crucially, no quantum concepts are assumed at the outset. In particular, we do *not* assume:

- complex-valued wavefunctions,
- probabilistic interpretation rules,
- canonical commutation relations,
- Hamiltonians defined from observables.

Instead, we impose only one universal requirement: *conservation of a quadratic structural norm.* We show that, under locality, this requirement alone enforces a Schrödinger-class continuum evolution as a structure-hiding form.

The logical structure of the paper is therefore:

$$\begin{aligned}\text{conservative wave-like dynamics} &\Longrightarrow \text{hidden two-channel rotational structure}\\ &\Longrightarrow \text{Schrödinger-type representation.}\end{aligned}$$

Minimal structural premise (JS–SH). Throughout this paper, the term *JS–SH structure* refers to a discrete system that explicitly realizes these minimal ingredients: (i) two-channel real internal states and (ii) local hub-mediated connectivity. Within such systems, the Schrödinger equation arises *necessarily* as a compressed continuum description, not as a postulate.

Table 1: Conceptual comparison between the conventional Schrödinger formulation and the structural interpretation adopted in this work.

Comparison item	**Standard Schrödinger formulation**	**Structural interpretation (this work)**
Logical status of Schrödinger equation	Fundamental postulate governing complex wavefunctions.	Effective, structure-hiding representation of an underlying real two-channel dynamics.
Direction of implication	Schrödinger equation $\Rightarrow$ physical interpretation.	Existence of Schrödinger-type dynamics $\Rightarrow$ presence of a hidden two-channel rotational structure.
Origin of complex phase (i)	Introduced axiomatically.	Canonical generator of rotations on real two-dimensional invariant sectors ($SO(2)$).
Role of JS–SH framework	Not applicable.	Explicit realization of the minimal structure implicit in Schrödinger-type equations; not claimed to be universal.
Physical constants ($\hbar$, m)	External parameters fixed by experiment.	Effective parameters determined by structural scales and coupling strengths.

2 Primitive JS–SH Structural Setup

2.1 JS-cells and discrete indexing

We consider a discrete set of structural units indexed by $i \in \mathcal{I}$. Each unit is referred to as a JS-*cell*. No spatial continuum is assumed at this stage; adjacency relations will be specified through SH-hubs.

Time is also taken to be discrete, indexed by $n \in \mathbb{Z}$. The discrete time step Δt is a structural parameter, not yet identified with any physical time unit.

2.2 Two-channel real internal state

At each JS-cell i and time step n, At this stage, the state is represented exclusively as a real two-channel vector.

$$\mathbf{a}_i^{(n)} = \begin{pmatrix} a_{i,1}^{(n)} \\ a_{i,2}^{(n)} \end{pmatrix} \in \mathbb{R}^2. \tag{1}$$

This two-channel structure is the *minimal* choice compatible with nontrivial internal dynamics while still permitting a conserved quadratic norm. A single real channel would allow only trivial sign changes, while higher-dimensional channels introduce redundant degrees of freedom.

At this stage:

- $\mathbf{a}_i^{(n)}$ is *not* a wavefunction,
- its components carry no probabilistic meaning,
- no complex numbers are introduced.

2.3 Structural norm

We define the local structural norm

$$\|\mathbf{a}_i^{(n)}\|^2 = \left(a_{i,1}^{(n)}\right)^2 + \left(a_{i,2}^{(n)}\right)^2, \tag{2}$$

and the global structural norm

$$\mathcal{N}^{(n)} = \sum_{i\in\mathcal{I}} \|\mathbf{a}_i^{(n)}\|^2. \tag{3}$$

The central physical requirement of this work is:

Structural conservation principle: $\mathcal{N}^{(n)}$ is invariant under time evolution.

This principle replaces all probabilistic and measurement axioms. It expresses the conservation of internal structural content under discrete evolution.

2.4 SH-hub connectivity (preview)

Interactions between neighboring JS-cells are mediated by SH-hubs. At this stage, we only assume:

- locality (only neighboring indices interact),
- linearity at leading order,
- symmetry under cell relabeling.

The explicit form of SH-mediated coupling will be constructed systematically in later sections.

Table 2: Structural ladder from JS–SH discrete requirements to Schrödinger-class dynamics.

Step	Structural requirement (JS–SH)	Forced mathematical consequence
1	**JS-cell as primitive state unit**: Each JS-cell carries a two-component real internal state $\mathbf{a}_i^{(n)} \in \mathbb{R}^2$.	Existence of a local real state vector $$\mathbf{a}_i^{(n)} = \begin{pmatrix} a_{i,1}^{(n)} \\ a_{i,2}^{(n)} \end{pmatrix},$$ with no complex structure assumed.
2	**SH-hub local connectivity**: Linear, nearest-neighbor mixing between JS-cells with finite range and locality.	Discrete Laplacian structure emerges in the ordered continuum limit, fixing the leading spatial operator as ∇^2 (up to symmetry-allowed higher-order corrections).
3	**Internal rotational symmetry**: Local invariance under channel rotations $\mathbf{a} \mapsto R(\theta)\mathbf{a}$ with $R(\theta) \in SO(2)$.	Unique nontrivial generator matrix $$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \mathbf{J}^2 = -\mathbf{I},$$ providing the structural origin of phase (complex notation introduced only as a convenience).
4	**Strict norm conservation**: $\sum_i \lVert \mathbf{a}_i^{(n)} \rVert^2 = \text{const}$ for all update steps n.	Discrete-time evolution must be exactly norm-preserving, hence expressible as an orthogonal update in the real representation: $$\mathbf{A}^{(n+1)} = \mathcal{U}\,\mathbf{A}^{(n)}, \qquad \mathcal{U}^{\mathsf{T}}\mathcal{U} = \mathbf{I},$$ with a generator of the form $$\mathcal{U} = \exp(\Delta t\,\mathbf{J}_{\mathrm{g}}\,\mathcal{G}),$$ where $\mathbf{J}_{\mathrm{g}}$ is the global generator assembled from the local 2×2 generator $\mathbf{J}$, and $\mathcal{G}$ is the symmetric SH-adjacency operator.
Result	**Ordered continuum limit** ($\Delta x, \Delta t \to 0$ with locality preserved)	**First-order conservative evolution law**: $$\dot{a}_i = \mathbf{J}[\omega_0\,a_i + \gamma_{\mathrm{SH}}\,(a_{i+1} - 2a_i + a_{i-1})],$$ i.e. the Schrödinger universality class, with ω_0, D fixed by SH-hub connectivity and structural scaling.

3 Structural Conservation and Internal $SO(2)$ Symmetry

In this section we determine the most general form of admissible local transformations acting on the JS-cell internal state that are compatible with the structural conservation principle. No continuum assumptions are made.

3.1 Local linear update: general form

We consider a single JS-cell and suppress the cell index i for notational clarity. The internal state at discrete time step n is

$$\mathbf{a}^{(n)} \in \mathbb{R}^2.$$

The most general *local linear* update compatible with discrete time evolution may be written as

$$\mathbf{a}^{(n+1)} = M\,\mathbf{a}^{(n)}, \tag{4}$$

where M is a real 2×2 matrix.

At this stage, M is completely unconstrained. Its form will be fixed by the structural conservation requirement.

3.2 Constraint from norm conservation

Structural conservation requires that the local contribution to the global norm remains invariant under the update:

$$\|\mathbf{a}^{(n+1)}\|^2 = \|\mathbf{a}^{(n)}\|^2 \quad \text{for all } \mathbf{a}^{(n)} \in \mathbb{R}^2. \tag{5}$$

Substituting (4) into (5) gives

$$(M\mathbf{a})^\mathsf{T}\,(M\mathbf{a}) = \mathbf{a}^\mathsf{T}\mathbf{a} \quad \text{for all } \mathbf{a}.$$

This condition is equivalent to

$$M^\mathsf{T} M = \mathbf{I}. \tag{6}$$

Hence, the admissible local update matrices M belong to the orthogonal group $O(2)$.

Interpretation. Norm preservation alone restricts the dynamics to orthogonal transformations of the internal two-channel state. No physical interpretation has been invoked.

3.3 Orientation and time direction

The orthogonal group $O(2)$ has two disconnected components:

- reflections, with $\det M = -1$,
- rotations, with $\det M = +1$.

Reflections reverse orientation and correspond to time-reversal-like operations at the internal level. A consistent forward-time evolution requires preservation of orientation.

We therefore restrict to

$$M \in SO(2). \tag{7}$$

This choice is not an additional physical postulate; it is a minimal consistency requirement for directed time evolution.

3.4 Parameterization of $SO(2)$

Every element of $SO(2)$ may be written as a rotation by an angle θ:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{8}$$

For small structural time steps Δt, we consider infinitesimal rotations with $\theta = \omega\,\Delta t$, where ω is a real parameter with dimensions of *inverse time*. At this stage, ω is purely structural and carries no physical interpretation.

Expanding (8) to first order in Δt yields

$$R(\omega\Delta t) = \mathbf{I} + \omega\Delta t \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} + \mathcal{O}(\Delta t^2). \tag{9}$$

3.5 Emergence of the generator matrix J

We define the unique 2×2 generator matrix

$$\mathbf{J} := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{10}$$

This matrix satisfies

$$\mathbf{J}^2 = -\mathbf{I}, \tag{11}$$

a purely algebraic identity.

Key point. The appearance of $\mathbf{J}$ is not an assumption. It is the *unique* generator of continuous norm-preserving, orientation-preserving transformations in a two-dimensional real internal space.

No complex numbers have been introduced. Equation (11) follows directly from real linear algebra.

3.6 Discrete-time evolution in generator form

The local update (4) may now be written as

$$\mathbf{a}^{(n+1)} = \exp(\omega\,\Delta t\,\mathbf{J})\;\mathbf{a}^{(n)}. \tag{12}$$

The sign of ω fixes the orientation of internal rotation and therefore the direction of time propagation. This choice is structural, not dynamical.

At this stage, the evolution:

- is linear,
- preserves the structural norm exactly,
- contains no reference to waves, particles, or probability,
- involves no complex-valued quantities.

3.7 Why two channels are minimal

It is instructive to note why the two-channel structure $\mathbb{R}^2$ is minimal.

- In $\mathbb{R}^1$, norm preservation allows only sign flips, yielding trivial dynamics.
- In $\mathbb{R}^2$, continuous norm-preserving rotations exist and admit a single nontrivial generator matrix $\mathbf{J}$.
- In $\mathbb{R}^N$ with $N > 2$, additional generators appear, leading to unnecessary internal degrees of freedom not supported by minimal structural assumptions.

Thus, the two-channel internal state is uniquely selected by the combination of minimality and structural conservation.

4 SH–Hub Coupling and Discrete Spatial Propagation

In this section we extend the local JS-cell dynamics derived previously to include interactions between neighboring cells. These interactions are mediated by SH-hubs and give rise to spatial propagation of internal structural phase.

The construction is entirely discrete and structural; no continuum notions such as fields or derivatives are assumed a priori.

4.1 Structural adjacency and SH-hubs

We assume that the set of JS-cells $\mathcal{I}$ is equipped with a notion of adjacency. For concreteness, we consider a one-dimensional ordered index set $i \in \mathbb{Z}$, though the construction generalizes straightforwardly to higher dimensions.

An SH-hub connects each JS-cell i to its nearest neighbors $i \pm 1$. The SH-hub does not store an independent state; it mediates linear exchange between internal states of adjacent cells.

Let γ_{SH} denote the SH-hub coupling strength. Its dimension is that of an inverse time and reflects the structural rate at which neighboring JS-cells exchange internal content.

4.2 Discrete Laplacian as the unique locality-preserving coupling

For a one-dimensional nearest-neighbor chain, the unique linear coupling compatible with: (i) locality, (ii) translation invariance, (iii) invariance under uniform shifts, is the discrete Laplacian (second difference). For a field $\mathbf{a}_i$ it is

$$(\Delta_d \mathbf{a})_i := \mathbf{a}_{i+1} - 2\mathbf{a}_i + \mathbf{a}_{i-1}. \tag{13}$$

No other linear nearest-neighbor operator satisfies all three conditions simultaneously.

4.3 Exact norm-preserving global update (strict conservation)

To implement *strict* global norm conservation, we construct an update that is exactly orthogonal in the real two-channel representation.

Let $\mathcal{I}$ be finite with N sites (e.g. periodic ring), and define the global state vector

$$\mathbf{A}^{(n)} := \begin{pmatrix} \mathbf{a}_1^{(n)} \\ \mathbf{a}_2^{(n)} \\ \vdots \\ \mathbf{a}_N^{(n)} \end{pmatrix} \in \mathbb{R}^{2N}, \qquad \mathcal{N}^{(n)} = \|\mathbf{A}^{(n)}\|^2. \tag{14}$$

Define the block-antisymmetric matrix

$$\mathbf{J}_{\mathrm{g}} := \mathbf{I}_N \otimes \mathbf{J}, \qquad \mathbf{J}_{\mathrm{g}}^{\top} = -\mathbf{J}_{\mathrm{g}}. \tag{15}$$

where $\mathbf{I}_N$ is the $N \times N$ identity and $\otimes$ is the Kronecker product.

Notation clarification (non-optional). We emphasize that $\mathbf{J} \in \mathbb{R}^{2\times 2}$ is the *local* internal generator acting on the two real channels of each JS-cell, whereas $\mathbf{J}_{\mathrm{g}} \in \mathbb{R}^{2N\times 2N}$ is the *lifted global* generator acting on the concatenated state vector $\mathbf{A}^{(n)}$ across all sites.

Next, define the discrete spatial generator $\mathcal{G}_d$ acting on site indices:

$$\mathcal{G}_d := \omega_0 \mathbf{I}_N + \gamma_{\mathrm{SH}}\,(-L). \tag{16}$$

where L is the standard symmetric graph Laplacian for the 1D nearest-neighbor ring:

$$(L\mathbf{u})_i := -u_{i+1} + 2u_i - u_{i-1}. \tag{17}$$

(Equivalently, $-L$ generates the second-difference structure (13).)

We lift $\mathcal{G}_d$ to the full two-channel space by

$$\mathcal{G} := \mathcal{G}_d \otimes \mathbf{I}_2.$$

Because $\mathcal{G}_d$ acts only on site indices and $\mathbf{J}_{\mathrm{g}}$ acts only on internal channels, they commute:

$$\mathbf{J}_{\mathrm{g}}\mathcal{G} = \mathcal{G}\mathbf{J}_{\mathrm{g}}.$$

Moreover, $\mathcal{G}$ is symmetric and $\mathbf{J}_{\mathrm{g}}$ is antisymmetric, hence $\mathbf{J}_{\mathrm{g}}\mathcal{G}$ is antisymmetric:

$$(\mathbf{J}_{\mathrm{g}}\mathcal{G})^{\top} = \mathcal{G}^{\top}\mathbf{J}_{\mathrm{g}}^{\top} = \mathcal{G}(-\mathbf{J}_{\mathrm{g}}) = -(\mathbf{J}_{\mathrm{g}}\mathcal{G}).$$

Therefore, the exact discrete-time update

$$\mathbf{A}^{(n+1)} = \mathcal{U}\,\mathbf{A}^{(n)}, \qquad \mathcal{U} := \exp(\Delta t\,\mathbf{J}_{\mathrm{g}}\mathcal{G})\,, \tag{18}$$

is strictly norm-preserving. Since $\mathbf{J}_{\mathrm{g}}\mathcal{G}$ is antisymmetric, $\mathcal{U}$ is orthogonal:

$$\mathcal{U}^{\top}\mathcal{U} = \mathbf{I}_{2N} \quad \Rightarrow \quad \mathcal{N}^{(n+1)} = \mathcal{N}^{(n)}.$$

Under complex repackaging, the action of the real generator $\mathbf{J}_{\mathrm{g}}$ is represented symbolically as multiplication by i, without introducing any new dynamical structure. This is the precise implementation of the structural conservation principle.

4.4 Local site form (first-order expansion)

To connect with the familiar neighbor-coupling form, expand (18) to first order in Δt:

$$\mathbf{A}^{(n+1)} = (\mathbf{I}_{2N} + \Delta t\,\mathbf{J}_{\mathrm{g}}\mathcal{G})\,\mathbf{A}^{(n)} + \mathcal{O}(\Delta t^2). \tag{19}$$

In site form, this yields for each i:

$$\mathbf{a}_i^{(n+1)} = \mathbf{a}_i^{(n)} + \Delta t\,\mathbf{J}\left[\omega_0\mathbf{a}_i^{(n)} + \gamma_{\mathrm{SH}}\left(\mathbf{a}_{i+1}^{(n)} - 2\mathbf{a}_i^{(n)} + \mathbf{a}_{i-1}^{(n)}\right)\right] + \mathcal{O}(\Delta t^2). \tag{20}$$

Thus, the intuitive nearest-neighbor update is recovered as the first-order structural expansion of the *exact* norm-preserving update. No inconsistency remains: strict conservation is built in at the discrete level, while (20) is its controlled small-Δt form.

4.5 Interpretation before continuum limit

At this stage, the theory describes:

- a discrete set of JS-cells,
- each carrying a two-channel real internal state,
- undergoing local internal rotation generated by $\mathbf{J}$,
- coupled to neighbors through conservative SH-hubs.

No continuum variables, derivatives, or complex amplitudes have been introduced. The structure is fully discrete and purely real.

Nevertheless, the form of (20) already suggests the emergence of a wave-like propagation mechanism, driven by internal phase rotation and spatial coupling.

5 Ordered Continuum Limit and Emergent Complex Representation

In this section we take the ordered continuum limit of the discrete JS–SH dynamics derived previously. We show that structural conservation and locality uniquely select a first-order-in-time continuum evolution. Only at this stage do we introduce a complex notation, purely as a compact representation of the underlying real two-channel state.

5.1 Discrete-to-continuum scaling

We associate to each JS-cell index i a spatial coordinate

$$x_i = i\,\Delta x, \tag{21}$$

where Δx is the structural spatial resolution. Similarly, the discrete time index n is associated with

$$t_n = n\,\Delta t. \tag{22}$$

The ordered continuum limit is defined by

$$\Delta x \to 0, \qquad \Delta t \to 0, \tag{23}$$

while keeping the coefficient

$$D := \gamma_{\mathrm{SH}}\,(\Delta x)^2 \tag{24}$$

finite.

The parameter D has dimensions of

$$[D] = \frac{(\mathrm{length})^2}{\mathrm{time}}.$$

5.2 Expansion of the discrete update

Starting from the discrete evolution equation (20), we write

$$\frac{\mathbf{a}_i^{(n+1)} - \mathbf{a}_i^{(n)}}{\Delta t} = \mathbf{J}\left[\omega_0 \mathbf{a}_i^{(n)} + \gamma_{\mathrm{SH}}\left(\mathbf{a}_{i+1}^{(n)} - 2\mathbf{a}_i^{(n)} + \mathbf{a}_{i-1}^{(n)}\right)\right] + \mathcal{O}(\Delta t). \tag{25}$$

In the ordered continuum limit, we identify

$$\mathbf{a}_i^{(n)} \longrightarrow \mathbf{a}(x,t), \tag{26}$$

and the discrete second difference becomes

$$\mathbf{a}_{i+1} - 2\mathbf{a}_i + \mathbf{a}_{i-1} = (\Delta x)^2\, \partial_x^2 \mathbf{a}(x,t) + \mathcal{O}(\Delta x^4). \tag{27}$$

Substituting into (25) yields

$$\partial_t \mathbf{a}(x,t) = \mathbf{J}\left[\omega_0\, \mathbf{a}(x,t) + D\, \partial_x^2 \mathbf{a}(x,t)\right]. \tag{28}$$

Equation (28) is the fundamental continuum evolution equation expressed entirely in real two-channel form.

5.3 Why first-order time evolution is forced

It is important to emphasize that the first-order time derivative in (28) is not an assumption.

Higher-order time derivatives would require either:

- nonlocal memory effects in the discrete update, or
- additional internal degrees of freedom per JS-cell.

Both possibilities violate the minimal JS–SH structural setup. Thus, first-order-in-time evolution is uniquely selected by:

- discrete locality,
- linearity at leading order,
- structural norm conservation.

5.4 Emergent complex representation

Equation (28) is written in terms of a real two-component field. For notational compactness, we now introduce a complex-valued field

$$\psi(x,t) := a_1(x,t) + \mathrm{i}\, a_2(x,t), \tag{29}$$

where i is introduced *by definition* as a symbol representing the action of the generator matrix $\mathbf{J}$:

$$\mathbf{J}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \longleftrightarrow \mathrm{i}\,(a_1 + \mathrm{i}a_2). \tag{30}$$

No new structure is introduced here; the complex notation is merely a shorthand for the already-derived real dynamics.

5.5 Continuum equation in compact form

In terms of $\psi(x,t)$, equation (28) takes the compact form

$$\partial_t \psi(x,t) = \mathrm{i}\left[\omega_0\, \psi(x,t) + D\, \partial_x^2 \psi(x,t)\right]. \tag{31}$$

At this stage:

- ψ is not postulated as a quantum wavefunction,
- i is not assumed as a primitive imaginary unit,
- the equation has been derived from discrete JS–SH structure alone.

Equation (31) represents the most general linear, local, conservative continuum dynamics compatible with the JS–SH framework.

6 The Equation Above Schrödinger: Structural Wave–Rotation Dynamics

6.1 Two kinds of “closure”: completed wavelength vs. unclosed phase

We will use the following structural distinction as the organizing principle of this paper:

- **Phase-closed (completed wavelength) sector:** the propagation process closes on itself after an integer number of boundary steps and restoration cycles. This sector produces stable standing patterns and time-harmonic eigenmodes.
- **Phase-open (unclosed) sector:** the propagation process does *not* close after a finite number of steps/cycles. This sector produces drift, beating, and envelope dynamics.

In the JS–SH picture, “wavelength” is not a primitive continuum parameter but an emergent counting object. For a macroscopic separation r, define $N_{\text{step}}(r)$ as the number of minimal boundary transitions along admissible JS paths, and define the effective wavelength by

$$\lambda_{\text{eff}}(r) \;:=\; \frac{r}{N_{\text{step}}(r)}. \tag{32}$$

A phase-closed sector is characterized by integer closure conditions on step counts and cycle counts; an open sector fails those integer closures and therefore requires a dynamical description for the residual.

6.2 Primitive discrete law: second-order structural wave on a JS adjacency

Let $u_n(t) \in \mathbb{R}$ be a primitive discrete state on a 1D ordered adjacency $n \in \mathbb{Z}$ with spacing $a > 0$. Define the discrete second difference operator

$$(\Delta_d u)_n \;:=\; \frac{u_{n+1} - 2u_n + u_{n-1}}{a^2}. \tag{33}$$

The equation-first structural wave law is

$$\ddot{u}_n(t) \;=\; c^2 (\Delta_d u)_n(t), \tag{34}$$

where $c > 0$ is a structural propagation ratio (in JS–SH, $c = R_{\text{JS}}/\tau_{\text{JS}}$).

6.3 Exact dispersion and the unavoidable quartic correction

Substituting the plane-wave mode $u_n(t) = \exp(i(kna - \omega t))$ into (34) yields the *exact* discrete dispersion relation

$$\omega^2(k) \;=\; \frac{4c^2}{a^2}\,\sin^2\!\Big(\frac{ka}{2}\Big). \tag{35}$$

In the long-wavelength regime $|ka| \ll 1$, we have

$$\omega^2(k) \;=\; c^2 k^2 \;-\; \frac{c^2 a^2}{12}\,k^4 \;+\; O(a^4 k^6), \tag{36}$$

and therefore the induced real-space effective equation for a smooth field $u(x,t)$ satisfying $u(na,t) = u_n(t)$ is

$$u_{tt} \;=\; c^2 u_{xx} \;+\; \frac{c^2 a^2}{12}\,u_{xxxx} \;+\; O(a^4). \tag{37}$$

The quartic term is not a phenomenological modification: it is the unique structural correction forced by the discrete second-difference stencil.

6.4 Why Schrödinger cannot be the primitive law

Equation (34) is **second-order in time** and lives on the primitive discrete adjacency. It yields both (i) phase-closed standing modes (integer closure) and (ii) phase-open beating/envelopes. A first-order-in-time Schrödinger-type equation can only describe a *subsector* of this hierarchy: it is an effective envelope/sector parametrization that compresses a deeper two-channel rotation plus a dispersion-selected spatial generator.

Accordingly, our goal is:

> Construct an explicit "equation above Schrödinger" that is structural and discrete-first, and show that the Schrödinger equation appears *below* it as a phase-sector/envelope representation.

7 From Primitive Wave to Two-Channel Structural Rotation

7.1 Two-channel decomposition: real carrier + quadrature partner

The primitive wave law (34) supports oscillatory carriers. To expose the structural origin of the imaginary unit, we introduce a *real two-channel* state

$$\Psi_n(t) \;:=\; \begin{pmatrix} q_n(t) \\ p_n(t) \end{pmatrix} \in \mathbb{R}^2, \qquad J \;:=\; \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad J^2 = -I. \tag{38}$$

The central claim (made representation-free in your JS–SH manuscript) is that norm-preserving linear dynamics on invariant 2-planes has the canonical rotational form

$$\dot{\Psi} \;=\; J\,\Omega\,\Psi, \tag{39}$$

for some real generator Ω (possibly an operator in space). Complex notation is *not* fundamental; it is a shorthand for the real J-rotation.

7.2 Structural adjacency operator and the long-wavelength fixed point

Let the discrete spatial generator be the nearest-neighbor Laplacian-like stencil on the adjacency:

$$(L\Psi)_n \;:=\; \Psi_{n+1} - 2\Psi_n + \Psi_{n-1}. \tag{40}$$

Stability and symmetry fix this quadratic-dispersion generator as the long-wavelength fixed point. Therefore, the structurally selected two-channel evolution is

$$\dot{\Psi}_n \;=\; J\Big(\omega_0\,\Psi_n \;+\; \gamma\,L\Psi_n\Big), \tag{41}$$

where ω_0 is a local structural phase-advance rate and γ is the adjacency coupling scale (in JS–SH language, $\gamma \sim \gamma_{\mathrm{SH}}$).

Equation (41) is the promised "equation above Schrödinger" at the discrete structural level: it is (i) discrete-first, (ii) norm-preserving by construction ($J^T = -J$), and (iii) it generates Schrödinger-type parametrizations only after a representation choice.

8 Continuum Induction and the Schrödinger Representation

8.1 Continuum induction from discrete adjacency

Assume an order-preserving embedding of indices into an ordered continuum, $x_n = \iota(n)$, with constant step $\Delta x := x_{n+1} - x_n$ (the structural point is: the discrete index is primitive; the continuum is a representation).

Define the continuum two-channel field $\Psi(x,t)$ by $\Psi(x_n,t) = \Psi_n(t)$. Then the discrete stencil satisfies the standard expansion

$$\Psi_{n+1} - 2\Psi_n + \Psi_{n-1} \;=\; (\Delta x)^2\,\partial_x^2\Psi(x,t) \;+\; \frac{(\Delta x)^4}{12}\,\partial_x^4\Psi(x,t) \;+\; O((\Delta x)^6). \tag{42}$$

Substituting (42) into the discrete master equation (41) yields the induced continuum "wave–rotation" law:

$$\partial_t\Psi(x,t) \;=\; J\Big(\omega_0(x)\,\Psi(x,t) \;+\; D\,\partial_x^2\Psi(x,t) \;+\; D_4\,\partial_x^4\Psi(x,t) \;+\; \cdots\Big), \tag{43}$$

where

$$D := \gamma(\Delta x)^2, \qquad D_4 := \gamma\frac{(\Delta x)^4}{12}. \tag{44}$$

The quartic correction here is the direct rotational analogue of the unavoidable k^4 term in the discrete wave derivation.

8.2 Complex notation as a compression: $J \mapsto i$

Define the complex field

$$\psi(x,t) \;:=\; q(x,t) + i\,p(x,t), \tag{45}$$

which is purely a notation for the real two-channel vector $\Psi = (q,p)^T$. Under this identification, multiplication by i corresponds exactly to the real rotation generator J:

$$J\Psi \;\longleftrightarrow\; i\psi. \tag{46}$$

Therefore, the continuum master law (43) becomes

$$\partial_t\psi \;=\; i\Big(\omega_0(x)\,\psi \;+\; D\,\partial_x^2\psi \;+\; D_4\,\partial_x^4\psi \;+\; \cdots\Big). \tag{47}$$

This is the JS–SH Schrödinger-type structure, but *still above* the textbook Schrödinger equation because (i) it is written in frequency units, and (ii) it carries a structurally fixed correction hierarchy.

8.3 Energy units: the structural conversion scale $\hbar_{\text{eff}}$

To express (47) in energy units, introduce the structural conversion constant

$$E \;:=\; \hbar_{\text{eff}}\Omega, \tag{48}$$

which is *not* a quantum postulate: it is the statement that a frequency scale can be expressed as an energy scale once the underlying structure fixes the conversion.

Multiplying (47) by $\hbar_{\text{eff}}$ gives

$$i\hbar_{\text{eff}}\,\partial_t\psi \;=\; \hbar_{\text{eff}}\omega_0(x)\,\psi \;+\; \hbar_{\text{eff}}D\,\partial_x^2\psi \;+\; \hbar_{\text{eff}}D_4\,\partial_x^4\psi \;+\; \cdots. \tag{49}$$

8.4 Recovering textbook Schrödinger as the leading envelope equation

Define the emergent potential (phase-bias energy)

$$V(x) \;:=\; \hbar_{\text{eff}}\omega_0(x), \tag{50}$$

and define the emergent effective mass by the *definition*

$$\frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}} \;:=\; \hbar_{\text{eff}}D. \tag{51}$$

(Here the sign convention is absorbed into D; the point is that ‘mass” is not imported, it labels the JS–SH diffusion resistance.)

Then the leading-order truncation of (49) yields

$$i\hbar_{\text{eff}}\,\partial_t\psi \;=\; -\frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}}\,\partial_x^2\psi \;+\; V(x)\psi \qquad \text{(Schrödinger limit).} \tag{52}$$

This is the standard Schrödinger equation, but here it is explicitly the **lowest rung** of the structural hierarchy: it is obtained by (i) choosing the two-channel rotational representation and (ii) truncating the structurally fixed correction series.

8.5 What the “above” equation predicts beyond Schrödinger

Keeping the next term in (49), we obtain the leading structural correction:

$$i\hbar_{\text{eff}}\,\partial_t\psi \;=\; -\frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}}\,\partial_x^2\psi \;+\; V(x)\psi \;+\; \eta\,\partial_x^4\psi \;+\; \cdots\,, \tag{53}$$

with a *fixed* coefficient

$$\eta \;:=\; \hbar_{\text{eff}}D_4 \;=\; \hbar_{\text{eff}}\gamma\frac{(\Delta x)^4}{12}. \tag{54}$$

Thus the discreteness does not merely *allow* corrections; it *forces* them with computable coefficients.

9 Phase-Closed vs Phase-Open: Why “All Schrödinger Solutions” Are Structural

9.1 Closed sector: integer step closure and standing eigenmodes

Let a propagation loop (or effective round-trip) have macroscopic length L. In the JS–SH operational definition, the phase advance is counted by boundary steps. Define $N_{\text{step}}(L)$ and the effective wavelength $\lambda_{\text{eff}}(L) = L/N_{\text{step}}(L)$ as in (32).

The **phase-closed** condition is the integer closure

$$N_{\text{step}}(L) \;\in\; \mathbb{Z}, \qquad \Delta\theta_{\text{loop}} \;=\; 2\pi\,m, \quad m \in \mathbb{Z}. \tag{55}$$

Equivalently, in wave-number language, closure is the quantization rule

$$k_{\text{eff}}\,L \;=\; 2\pi m, \qquad k_{\text{eff}} \;:=\; \frac{2\pi}{\lambda_{\text{eff}}(L)}. \tag{56}$$

In the closed sector, solutions are time-harmonic carriers:

$$\psi(x,t) = \phi_m(x)\,e^{-i\Omega_m t}, \tag{57}$$

and Schrödinger stationary states appear simply because the structural wave–rotation admits stable eigenmodes when the step-closure is satisfied.

9.2 Open sector: residual mismatch as envelope dynamics

If the propagation does *not* close,

$$\Delta\theta_{\text{loop}} \;=\; 2\pi m \;+\; \delta, \qquad \delta \neq 0, \tag{58}$$

then the system cannot remain in a single stationary carrier. Instead, the mismatch produces slow drift / beating. This is precisely the regime where a first-order-in-time envelope equation becomes the correct macroscopic description.

Let the full two-channel field be a modulated carrier:

$$\psi(x,t) \;=\; A(x,t)\, e^{-i\Omega_0 t}, \qquad \text{with } A \text{ slowly varying compared to } e^{-i\Omega_0 t}. \tag{59}$$

Substituting (59) into the master law (47), and separating the fast carrier Ω_0 from the slow residual, yields the envelope equation at leading order:

$$i\,\partial_t A \;=\; \Big(\omega_0(x) - \Omega_0\Big)A \;+\; D\,\partial_x^2 A \;+\; \cdots. \tag{60}$$

Multiplying by $\hbar_{\text{eff}}$ and applying (50)–(51) gives the Schrödinger form for A. Hence, the Schrödinger equation is the **phase-open residual equation** governing how the structure repairs non-closure at the macroscopic scale.

9.3 Why this matches your "completed wavelength vs not" idea

Your organizing idea can now be stated cleanly:

- If wavelength is *completed* (closure), the dynamics locks into eigenmodes and time-harmonic solutions.
- If wavelength is *not completed* (non-closure), the mismatch δ forces a slow evolution of the envelope, whose universal form is Schrödinger-type because the underlying law is a two-channel rotation with a stability-selected Laplacian generator.

Thus, "all Schrödinger solutions" in the JS–SH framework are not axioms; they are structural envelopes generated by (i) discrete adjacency, (ii) two-channel rotation, and (iii) closure vs non-closure in boundary-step counting.

9.4 Wavelength as geometry: inserting $\lambda_{\text{eff}}(r)$ into the phase accounting

Because $\lambda_{\text{eff}}(r) = r/N_{\text{step}}(r)$, the effective wave-number is

$$k_{\text{eff}}(r) \;=\; \frac{2\pi}{\lambda_{\text{eff}}(r)} \;=\; \frac{2\pi\, N_{\text{step}}(r)}{r}. \tag{61}$$

Therefore closure/non-closure is controlled directly by the step grammar and the local step inflation. In curved/defect-rich regions the step count inflates, which shifts k_{eff} and hence shifts the closure condition. This is the structural origin of "wavelength modulation" without changing the invariant propagation ratio c.

10 Falsifiability: what would refute the JS–SH Schrödinger origin

The hierarchy predicts:

- A universal quartic correction structure fixed by the discrete stencil (hence fixed coefficient once (Δx) is fixed).
- Closure-dependent selection of stationary eigenmodes vs open-sector envelope evolution.
- Geometry-induced modulation of λ_{eff} via step inflation, hence observable phase anomalies in multi-path systems.

Any observation that requires (i) first-order-in-time Schrödinger as primitive with no underlying two-channel rotation, or (ii) removal of the quartic correction within the nearest-neighbor discrete class, or (iii) wavelength modulation with N_{step} *not* entering phase accounting, would refute the structural origin thesis.

11 Final Synthesis: The Structural Ladder and the Position of Schrödinger

11.1 The ladder of descriptions

We can now state the "ladder" explicitly.

Level 0 (primitive, discrete-first): Discrete adjacency + second difference is the structural generator. The primitive wave law is (34), with exact dispersion (35).

Level 1 (structural two-channel rotation, still discrete-first): Norm preservation and locality force a two-channel rotational law

$$\dot{\Psi}_n \;=\; J\Big(\omega_0\,\Psi_n + \gamma\,(\Psi_{n+1} - 2\Psi_n + \Psi_{n-1})\Big), \tag{62}$$

which is the "equation above Schrödinger" in its cleanest discrete form.

Level 2 (continuum-induced, correction hierarchy visible): Under an ordered embedding with step Δx, the continuum master equation is

$$\partial_t\psi \;=\; i\Big(\omega_0(x)\psi + D\,\partial_x^2\psi + D_4\,\partial_x^4\psi + \cdots\Big), \tag{63}$$

with structurally fixed coefficients $D = \gamma(\Delta x)^2$ and $D_4 = \gamma(\Delta x)^4/12$.

Level 3 (energy units; Schrödinger as lowest truncation): Introducing $\hbar_{\text{eff}}$ as the structural frequency-to-energy conversion and defining V and m_{eff} by (50)–(51), the leading truncation is

$$i\hbar_{\text{eff}}\,\partial_t\psi \;=\; -\frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}}\,\partial_x^2\psi + V(x)\psi, \tag{64}$$

i.e. the textbook Schrödinger equation, but now explicitly *below* the structural master law.

11.2 One sentence conclusion

> The Schrödinger equation is not a primitive axiom: it is the lowest-order, phase-sector/envelope representation of a deeper discrete wave–rotation dynamics on JS–SH adjacency, whose correction hierarchy and closure selection are fixed by step-counting geometry.

Data and Reproducibility Note (structural, not interpretational)

All objects introduced above are structural:

- J is the real rotation generator on a two-channel invariant sector.
- ω_0 and γ are structural phase and adjacency coupling parameters.
- $\lambda_{\text{eff}}(r) = r/N_{\text{step}}(r)$ is operationally defined by admissible boundary-step counting.
- $\hbar_{\text{eff}}$ is a conversion scale between frequency and energy once the substrate fixes the unit system.

No probabilistic axiom, no Born rule, and no continuum smoothness as a primitive assumption is required for the operator-level emergence.

12 JS–SH Unhiding of the Schrödinger Equation

12.1 Schrödinger equation as a structure-hiding continuum form

Sections 2–5 established the fundamental JS–SH dynamics entirely in terms of real two-channel JS-cell states $a_i \in \mathbb{R}^2$, SH-hub mediated locality, the generator matrix J, and strict structural norm conservation. The resulting ordered continuum equation was obtained as

$$\partial_t a(x,t) = J\Big[\omega_0\, a(x,t) + D\,\partial_x^2 a(x,t)\Big]\,, \qquad D = \gamma_{\mathrm{SH}}(\Delta x)^2, \tag{65}$$

which is written purely in real two-channel form.

When repackaged as $\psi := a_1 + \mathrm{i}a_2$, equation (65) becomes

$$\partial_t \psi = \mathrm{i}\Big[\omega_0 \psi + D\,\partial_x^2 \psi\Big]\,. \tag{66}$$

Equation (66) is mathematically identical to a Schrödinger-type evolution. However, in the present framework this equation is *not fundamental*: it is a compact representation that hides the underlying JS–SH structural flow.

12.2 Unhiding principle: site-wise structural continuity

The Schrödinger form (66) encodes only global norm conservation. JS–SH dynamics satisfies a stronger requirement: *structural norm conservation must hold locally at each JS-cell via explicit SH-mediated transport.*

Define the local structural density

$$\rho_i(t) := \|a_i(t)\|^2 = a_i^{\mathsf{T}}(t) a_i(t). \tag{67}$$

JS–SH requires that there exist antisymmetric SH-edge fluxes $\mathcal{J}_{ij} = -\mathcal{J}_{ji}$ such that

$$\boxed{\frac{d}{dt}\rho_i(t) = \sum_{j\in\mathrm{SH}(i)} \mathcal{J}_{ij}(t)} \tag{68}$$

for every JS-cell i.

12.3 Canonical JS–SH flux factorization

Let the SH-hub coupling be symmetric, $\omega_{ij} = \omega_{ji} \geq 0$, consistent with the JS–SH graph Laplacian structure used in equations (13)–(17) of the main text. JS–SH closes the continuity equation (68) by the unique bilinear form induced by the generator matrix J:

$$\boxed{\mathcal{J}_{ij} = 2\,\omega_{ij}\, a_i^{\mathsf{T}} \mathbf{J}\, a_j} \tag{69}$$

where $J^T = -J$ ensures $\mathcal{J}_{ij} = -\mathcal{J}_{ji}$.

The scalar $a_i^{\mathsf{T}} J a_j$ is the oriented area in the internal two-channel plane and measures the directed structural phase exchange between neighboring JS-cells.

12.4 Exact compatibility with JS–SH evolution

Using the discrete JS–SH update (Section 4),

$$\dot{a}_i = J\left[\omega_0\, a_i + \sum_{j\in\mathrm{SH}(i)} \omega_{ij}\,(a_j - a_i)\right], \tag{70}$$

one computes

$$\frac{d}{dt}\rho_i = 2a_i^\top \dot{a}_i = 2 \sum_{j\in \mathrm{SH}(i)} \omega_{ij}\, a_i^\top J a_j, \tag{71}$$

which reproduces (68) with flux (69). Thus, the JS–SH evolution law satisfies exact site-wise structural continuity.

12.5 Structural meaning of the emergent continuum equation

Equation (66) therefore represents a *structure-hiding form* of the JS–SH dynamics:

- the two-channel internal structure is compressed into a single complex symbol,
- the generator matrix J is hidden as the imaginary unit i,
- SH-mediated phase exchange is summarized by the Laplacian term.

The *unhidden* dynamics is given by (65) together with the local continuity law (68) and the canonical flux (69). Any evolution that cannot be decomposed in this manner is not JS–SH admissible, even if it can be written in a compact first-order conservative form.

12.6 Position of the emergent continuum equation in the JS–SH hierarchy

We may therefore summarize the logical hierarchy as

$$\boxed{\text{JS–SH discrete structure} \implies \text{real two-channel continuum dynamics.}} \tag{72}$$

In this work, the emergent continuum equation is not a starting axiom but a *derived, subordinate representation* of a more fundamental JS–SH structural law.

13 JS–SH Unhiding Equation: Forcing the Hidden Structure Behind Schrödinger Evolution

13.1 Motivation: Schrödinger as a compressed (structure-hiding) description

In the continuum, Schrödinger-type evolution can appear as a closed, compact description that does not explicitly expose the underlying JS–SH connectivity. To *unhide* the structure, we impose a stronger requirement than global norm conservation: *site-wise continuity (local conservation) with an SH-mediated flux decomposition.* This yields an additional *closure/compatibility equation* not contained in the standard Schrödinger form.

13.2 JS–cell state space and the canonical phase generator

A JS–cell at site i carries a *two-channel real state*

$$a_i(t) \in \mathbb{R}^2, \qquad \rho_i(t) := \|a_i(t)\|^2 = a_i(t)^\top a_i(t). \tag{73}$$

The unique (up to sign) generator of orientation-preserving norm-preserving internal evolution in $\mathbb{R}^2$ is the antisymmetric matrix

$$J := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad J^\top = -J, \qquad J^2 = -I. \tag{74}$$

When convenient, one may identify (a_1, a_2) with $\psi = a_1 + ia_2$, where multiplication by i corresponds to the linear action of J. This is a *representation choice*; the real two-channel JS–SH dynamics is primary.

13.3 SH connectivity and the discrete transport operator (main-text convention)

Let $\mathrm{SH}(i)$ denote the SH-neighborhood of site i (nearest neighbors in the simplest case). In the main-text JS–SH convention, SH coupling is encoded by *symmetric* edge weights

$$\omega_{ij} = \omega_{ji} \geq 0, \qquad \omega_{ij} = 0 \text{ if } j \notin \mathrm{SH}(i), \tag{75}$$

and the SH transport operator (graph Laplacian form) is defined by

$$(\mathcal{L}_{\mathrm{SH}} a)_i := \sum_{j \in \mathrm{SH}(i)} \omega_{ij}\,(a_j - a_i). \tag{76}$$

For a translationally homogeneous 1D chain with spacing Δx, the minimal nearest-neighbor realization is

$$\omega_{i,i\pm 1} = \gamma_{\mathrm{SH}}, \qquad \omega_{ij} = 0 \text{ otherwise}, \tag{77}$$

so that

$$(\mathcal{L}_{\mathrm{SH}} a)_i = \gamma_{\mathrm{SH}}\,(a_{i+1} - 2a_i + a_{i-1}), \tag{78}$$

and the ordered continuum coefficient is

$$D = \gamma_{\mathrm{SH}} (\Delta x)^2 \qquad \text{(main-text continuum identification)}. \tag{79}$$

13.4 The JS–SH Unhiding (Closure) Equation: local norm continuity

Key principle. Global conservation $\frac{d}{dt}\sum_i \rho_i = 0$ does *not* expose how the conservation is realized through the JS–SH discrete structure. JS–SH imposes the stronger requirement that conservation holds *site-by-site* via SH-mediated pairwise fluxes.

Unhiding equation (discrete continuity + SH flux decomposition). We require the existence of antisymmetric edge fluxes $\mathcal{J}_{ij} = -\mathcal{J}_{ji}$ such that for every site i,

$$\boxed{\frac{d}{dt}\rho_i(t) = \sum_{j \in \mathrm{SH}(i)} \mathcal{J}_{ij}(t)} \qquad \text{with} \qquad \boxed{\mathcal{J}_{ij}(t) = -\mathcal{J}_{ji}(t)}. \tag{80}$$

JS–SH further *closes* the flux in terms of the primitive JS variables by the canonical bilinear form induced by J:

$$\boxed{\mathcal{J}_{ij}(t) = 2\,\gamma_{\mathrm{SH}}\, a_i(t)^{\mathsf{T}} J\, a_j(t).} \tag{81}$$

Because γ_{SH} is symmetric under $i \leftrightarrow j$ and $a_i^{\mathsf{T}} J a_j = -a_j^{\mathsf{T}} J a_i$, equation (81) automatically enforces $\mathcal{J}_{ij} = -\mathcal{J}_{ji}$.

Interpretation. The scalar $a_i^{\mathsf{T}} J a_j$ is the oriented area (symplectic form) in the two-channel plane, and thus measures the *directed phase-exchange capacity* between neighboring JS–cells across SH links. Condition (81) states that all norm transport must factor through these SH exchanges; any evolution whose norm conservation cannot be decomposed in this way is *inadmissible* as a JS–SH dynamics.

13.5 Compatible dynamics: the minimal JS–SH evolution law (main-text form)

The minimal linear, local JS–SH evolution consistent with the unhiding equation is

$$\boxed{\frac{d}{dt} a_i(t) = J[\omega_0\, a_i(t) + (\mathcal{L}_{\mathrm{SH}} a(t))_i]\,,} \tag{82}$$

where ω_0 is the on-site (uniform) internal rotation frequency (main-text notation), and $\mathcal{L}_{\mathrm{SH}}$ is given by (76).

Proposition (exact site-wise continuity). If $a(t)$ evolves by (82) with $\mathcal{L}_{\mathrm{SH}}$ defined by (76), then $\rho_i = \|a_i\|^2$ satisfies the continuity law (80) with flux (81).

Sketch of verification. Compute

$$
\begin{aligned}
\frac{d}{dt}\rho_i &= \frac{d}{dt}(a_i^{\top} a_i) = 2a_i^{\top}\frac{d}{dt}a_i = 2\,a_i^{\top} J[\omega_0 a_i + (\mathcal{L}_{\mathrm{SH}}a)_i] \\
&= 2\,a_i^{\top} J(\mathcal{L}_{\mathrm{SH}}a)_i \qquad (\text{since } a_i^{\top} J a_i = 0) \\
&= 2\sum_{j\in \mathrm{SH}(i)} \omega_{ij}\, a_i^{\top} J\,(a_j - a_i) = 2\sum_{j\in \mathrm{SH}(i)} \omega_{ij}\, a_i^{\top} J\, a_j,
\end{aligned}
\tag{83}
$$

which is precisely (80) with the identification $\mathcal{J}_{ij} := 2\omega_{ij}\, a_i^{\top} J a_j$. In the nearest-neighbor specialization (77), this reduces to the main-text flux normalization (81).

13.6 Continuum limit: Schrödinger-type form with explicit structural origin

On a regular lattice with spacing Δx and homogeneous coupling, $\mathcal{L}_{\mathrm{SH}}$ converges to $\gamma_{\mathrm{SH}}(\Delta x)^2\nabla^2$ in the ordered continuum limit. Let $a_i(t) \approx a(x,t)$ with $x = i\Delta x$. Then (82) yields

$$
\partial_t a(x,t) = J\Big[\omega_0\, a(x,t) + D\,\nabla^2 a(x,t)\Big], \qquad D = \gamma_{\mathrm{SH}}(\Delta x)^2,
\tag{84}
$$

which is the main-text real two-channel continuum JS–SH equation.

Under the identification $\psi = a_1 + ia_2$ (so that $Ja \leftrightarrow i\psi$), equation (84) becomes

$$
\partial_t \psi(x,t) = i\Big[\omega_0\, \psi(x,t) + D\,\nabla^2 \psi(x,t)\Big].
\tag{85}
$$

Thus the Schrödinger-type equation appears as a *compressed* continuum representation of a JS–SH dynamics whose hidden content is made explicit by the unhiding (closure) equation (80)–(81).

13.7 Why this is not a mere re-expression: a structural admissibility test

Standard Schrödinger form (85) alone does not specify whether a given unitary evolution can be realized as an SH-mediated exchange of local norm. JS–SH supplies an *admissibility criterion*:

> *A candidate evolution is JS–SH-admissible only if its norm transport admits a decomposition into SH-edge antisymmetric fluxes of the canonical form* $\mathcal{J}_{ij} = 2\,\omega_{ij}\, a_i^{\top} J a_j$ *supported solely on* $j \in \mathrm{SH}(i)$.

This additional requirement provides an explicit structural admissibility condition: it reinstates the JS–SH connectivity as a concrete realization criterion, thereby characterizing a specific subclass of unitary wave dynamics without asserting universality or necessity.

13.8 Structural interpretation of Schrödinger-type dynamics

Schrödinger-type equations appear in a wide variety of physical and mathematical settings, including contexts not explicitly formulated on JS–SH lattices. In this subsection, we do *not* claim universality or necessity. Instead, we clarify what structural features are *unavoidably present* whenever a Schrödinger-type description is valid.

In particular, the JS–SH framework is sufficient but not necessary: Schrödinger-type equations may arise from other structures, but whenever they do, they necessarily encode an underlying real two-channel rotational sector with continuity-enforced flux.

Representation-free setting. Let $\mathcal{H}$ be a real Hilbert space with a positive-definite inner product $\langle \cdot, \cdot \rangle$, and consider a linear evolution

$$\dot{u}(t) = \mathcal{K}u(t), \qquad u(t) \in \mathcal{H}, \tag{86}$$

that preserves the norm,

$$\frac{d}{dt}\|u(t)\|^2 = 0. \tag{87}$$

Such flows are generated by skew-adjoint operators and therefore correspond to rotations on real state space.

Two-channel rotational sectors. For any initial condition $u(0)$, the invariant subspace generated by $\{u(0), \mathcal{K}u(0)\}$ is at most two-dimensional. On each such invariant real 2-plane, the dynamics can be written in canonical form

$$\dot{u} = J\,\Omega\,u, \qquad J^{\mathsf{T}} = -J, \quad J^2 = -I, \tag{88}$$

with J unique up to orientation. Thus, the appearance of a complex phase factor in Schrödinger-type equations reflects the standard normal form of rotations on real two-dimensional sectors, rather than an independent axiom.

Local density and flux structure. When the conserved quantity admits a *local, positive-definite scalar density* satisfying a continuity equation, the associated current on each two-channel sector can be written, up to divergence-free redefinitions, in the bilinear antisymmetric form

$$\mathcal{J} \sim a^{\mathsf{T}} J \nabla a. \tag{89}$$

This is the same structural form that appears explicitly in the JS–SH flux decomposition.

Interpretation. These observations do *not* imply that all norm-preserving or locally conserved systems reduce to Schrödinger dynamics, nor that JS–SH provides a universal origin for such equations. Rather, they show that whenever a Schrödinger-type equation is an appropriate effective description, it necessarily compresses an underlying real two-channel rotational structure together with a continuity-enforced flux. The JS–SH framework makes this hidden structure explicit, whereas the standard Schrödinger representation leaves it implicit.

14 Identification with the Schrödinger Equation

In this final section we show that the continuum equation derived purely from JS–SH discrete structure admits a Schrödinger-type representation. No claim of universality or necessity is made beyond the JS–SH structural class. Crucially, the Schrödinger equation is *not assumed*; it is recognized as a non-unique, structure-hiding parametrization of the already-derived dynamics.

14.1 Recap: the structurally derived continuum equation

From Section 5, the continuum evolution equation obtained directly from JS–SH discrete dynamics is

$$\mathrm{i}\hbar_{\mathrm{eff}}\partial_t\psi = -\hbar_{\mathrm{eff}}\omega_0\,\psi - \hbar_{\mathrm{eff}}D\,\partial_x^2\psi. \tag{90}$$

where:

- $\psi(x,t)$ is a compact notation for a real two-channel field,
- i represents the phase generator induced by $\mathbf{J}$,

- ω_0 is the local structural phase-advance rate,
- $D = \gamma_{\mathrm{SH}}(\Delta x)^2$ is fixed by JS–SH connectivity.

Equation (66) is the *only* linear, local, norm-preserving continuum limit compatible with the JS–SH assumptions.

14.2 Energy units and the emergence of $\hbar_{\mathrm{eff}}$

Equation (66) is written in frequency units. To express the dynamics in energy units, we introduce a structural conversion constant $\hbar_{\mathrm{eff}}$ defined by

$$E := \hbar_{\mathrm{eff}}\,\omega_0. \tag{91}$$

This is not a quantum postulate. It is simply the statement that any frequency can be expressed in energy units once a conversion scale is fixed by the underlying structure.

Multiplying (66) by $\hbar_{\mathrm{eff}}$ gives

$$\mathrm{i}\hbar_{\mathrm{eff}}\partial_t\psi = -\hbar_{\mathrm{eff}}\omega_0\,\psi - \hbar_{\mathrm{eff}}D\,\partial_x^2\psi. \tag{92}$$

14.3 Emergent mass and potential

We now introduce standard energy-scale symbols purely as *definitions*, not as physical postulates.

The on-site structural rotation rate ω_0 is taken to be uniform (constant) throughout this work, consistent with the JS–SH derivation in the preceding sections.

We therefore define

$$V := -\hbar_{\mathrm{eff}}\omega_0 \tag{50}$$

$$\frac{\hbar_{\mathrm{eff}}^2}{2m_{\mathrm{eff}}} := \hbar_{\mathrm{eff}}D \tag{51}$$

Here:

- V is the emergent (uniform) potential associated with the on-site JS–SH phase bias,
- m_{eff} is the emergent effective mass encoding resistance to structural phase diffusion.

No mechanical or Newtonian interpretation is assumed. Both quantities arise solely as parameter redefinitions of the structurally derived continuum equation.

14.4 Final form

Substituting (50)–(51) into (92), we obtain

$$\mathrm{i}\hbar_{\mathrm{eff}}\partial_t\psi = -\frac{\hbar_{\mathrm{eff}}^2}{2m_{\mathrm{eff}}}\,\partial_x^2\psi + V\psi. \tag{93}$$

Equation (93) is mathematically identical to the Schrödinger equation.

However, in this work it is not a postulate: it is the inevitable continuum expression of JS–SH discrete dynamics.

14.5 Why this is not circular

The derivation avoids circularity for the following reasons:

- The primitive state is real and two-dimensional; complex numbers appear only as notation.
- Norm conservation is imposed structurally, not via the Born rule.
- The generator matrix $\mathbf{J}$ precedes i conceptually.
- $\hbar_{\text{eff}}$ is defined as a structural conversion scale, not imported as a quantum axiom.
- Mass and potential are introduced *after* the equation is derived.

Thus, no quantum axiom is imported at any stage.

14.6 Physical interpretation (beginner-friendly)

In the JS–SH framework:

- A "wavefunction" is a bookkeeping device for two coupled real structural channels.
- Probability conservation is structural norm conservation.
- Phase evolution is internal rotation of JS-cell states.
- Quantum dynamics is phase transport through SH-hub connectivity.

The Schrödinger equation is therefore not mysterious; it is the simplest continuum law describing conservative phase flow on a discrete structural substrate.

15 Extension: Structural utility in higher dimensions

Scope and non-circularity. This section is *not* used anywhere in the derivation of the Schrödinger-class evolution obtained in the preceding sections. All results of the main derivation close within the 2D (two-channel) JS–SH setting. Here we only record how the same JS–SH *structural principles* (two-channel cell state, SH-mediated locality, and norm-preserving updates) can be extended to higher-dimensional assemblies once the 2D case is fixed. Accordingly, nothing in this section is an additional assumption; it is a post-derivation structural *extension.*

15.1 Why the primitive state is two-channel, even in higher-dimensional space

The "two-channel" requirement refers to the *internal* cell state $\mathbf{a}_i^{(n)} \in \mathbb{R}^2$, not to the dimension of physical space. In particular, increasing the spatial dimension from $d = 1$ to $d = 2, 3, \ldots$ does not change the minimal internal structure needed for conservative phase-capable dynamics: the smallest nontrivial continuous symmetry that preserves a non-degenerate quadratic norm is the internal rotation group $SO(2)$ acting on $\mathbb{R}^2$. Its generator matrix

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \mathbf{J}^2 = -\mathbf{I}_2,$$

is therefore the unique minimal "phase" structure independent of the embedding spatial dimension. Thus the internal two-channel state is a *structural minimum* for conservative dynamics with a phase-like degree of freedom, while spatial dimension d is an *external* assembly choice.

15.2 JS–SH adjacency in d dimensions

Let the JS-cells occupy a regular d-dimensional lattice with spacing Δx (or a locally ordered graph that admits an ordered-neighborhood limit). Denote the cell index by $i \in \mathcal{I} \subset \mathbb{Z}^d$ and let $\mathcal{N}(i)$ be the set of nearest neighbors of i. The SH-hub rule "local mixing across neighbors" is implemented by an update whose generator depends only on adjacent cells,

$$\mathbf{a}_i^{(n+1)} = \sum_{j \in \mathcal{N}(i) \cup \{i\}} \mathbf{M}_{ij}\, \mathbf{a}_j^{(n)},$$

where the block matrices $\mathbf{M}_{ij} \in \mathbb{R}^{2\times 2}$ are constrained by: (i) locality ($\mathbf{M}_{ij} = 0$ unless $j \in \mathcal{N}(i) \cup \{i\}$), (ii) internal channel-rotation covariance (commutation with $SO(2)$ rotations), and (iii) global norm preservation.

Under the same ordered-neighborhood limit used in the main text, the leading spatial operator forced by adjacency is the d-dimensional discrete Laplacian,

$$(\Delta_d \psi)_i = \frac{1}{(\Delta x)^2} \left(\sum_{j \in \mathcal{N}(i)} \psi_j - |\mathcal{N}(i)|\, \psi_i \right),$$

which converges to ∇^2 as $\Delta x \to 0$ in an ordered continuum limit. Here ψ denotes the complex repackaging $\psi := a_1 + \mathrm{i} a_2$ (a definition), so the Laplacian acts on the repackaged field componentwise.

15.3 Conservative update and the same universality class in d dimensions

Norm preservation of the global discrete state, $\sum_i \|\mathbf{a}_i^{(n)}\|^2 = \text{const}$, enforces that the linear update is an orthogonal transformation in the real two-channel representation, and therefore becomes unitary under complex repackaging:

$$\psi^{(n+1)} = U_d\, \psi^{(n)}, \qquad U_d^{\dagger} U_d = I.$$

For sufficiently small Δt, one may parameterize

$$U_d = \exp\big(-\mathrm{i}\, \Delta t\, H_d\big), \qquad H_d = H_d^{\dagger},$$

where H_d is a discrete Hermitian generator constructed from local SH-hub adjacency (e.g. a function of Δ_d plus local bias terms).

In the ordered continuum limit, the same universality class emerges, now with a d-dimensional Laplacian:

$$\partial_t \psi(t, x) = -\mathrm{i}\Big(\Omega(x) + D\nabla^2 + \cdots\Big)\psi(t, x), \qquad x \in \mathbb{R}^d.$$

Importantly, this does not strengthen the main claim; it merely shows that *once* the 2D internal structure and conservative locality are fixed, spatial dimension enters only through the adjacency operator (Laplacian class).

15.4 What "higher-dimensional utility" means operationally

Operationally, the higher-dimensional extension provides:

- **Multi-dimensional locality without new postulates:** the SH-hub adjacency rule uniquely selects the Laplacian class in any dimension d (or any locally ordered graph admitting a Laplacian limit).
- **A clean separation of roles:** the internal two-channel structure explains the existence of a phase generator (matrix $\mathbf{J}$), while the external spatial assembly (d) controls dispersion geometry through ∇^2.

- **A direct bridge to testbeds:** higher-dimensional assemblies are the natural setting for scattering, wavepacket spreading, and interference geometry. These effects follow from the same discrete conservative update and the same continuum limit criterion, rather than from quantum postulates.

Minimal takeaway. The main derivation establishes the Schrödinger universality class from JS–SH structural requirements. This section only records that the same requirements extend naturally to higher-dimensional adjacency, with no additional assumptions and no feedback into the derivation.

16 Conclusion

We have derived the Schrödinger equation from first principles within the JS–SH discrete framework. The derivation uses only:

1. two-channel real JS-cell states,
2. SH-hub mediated locality,
3. internal $SO(2)$ symmetry (generator matrix $\mathbf{J}$),
4. structural norm conservation.

All familiar quantum symbols emerge as secondary parametrizations. Nothing is assumed from standard quantum mechanics.

This completes the derivation.

A Optional Numerical Sanity Check (Non-essential)

This appendix provides an optional numerical sanity check illustrating that the discrete JS–SH update rule yields a norm-preserving (unitary) evolution and reproduces Schrödinger-class dispersion in the continuum limit. No empirical or experimental data are used; all inputs are intrinsic structural parameters of the JS–SH framework.

A.1 Boxed code (Python)

File: `jssh_sch_sanity_min.py`

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-

"""
Minimal JS--SH sanity check.
- Discrete JS-cells on a 1D periodic ring
- SH-hub mediated Laplacian coupling
- Norm-preserving unitary update

NOTE (sign convention):
The matrix returned by build_laplacian() below implements the second-difference operator
(u_{i+1} - 2u_i + u_{i-1}), which equals (- graph-Laplacian) in the main-text convention.
Accordingly, H is constructed with (-L) so that H = omega0*I + D*(graph-Laplacian) as desired.
"""

import argparse
import numpy as np
```

```python


def build_laplacian(N: int) -> np.ndarray:
    # This returns the second-difference operator on a periodic ring:
    # (L u)_i = u_{i+1} - 2u_i + u_{i-1}
    L = -2.0 * np.eye(N, dtype=float)
    for i in range(N):
        L[i, (i - 1) % N] = 1.0
        L[i, (i + 1) % N] = 1.0
    return L


def norm2(psi: np.ndarray) -> float:
    return float(np.vdot(psi, psi).real)


def main() -> None:
    ap = argparse.ArgumentParser()
    ap.add_argument("--N", type=int, default=128)
    ap.add_argument("--dt", type=float, default=0.05)
    ap.add_argument("--D", type=float, default=0.6)
    ap.add_argument("--omega0", type=float, default=0.2)
    ap.add_argument("--steps", type=int, default=400)
    args = ap.parse_args()

    N = args.N
    dt = args.dt
    D = args.D
    omega0 = args.omega0
    steps = args.steps

    L = build_laplacian(N)
    H = omega0 * np.eye(N) + D * (-L)

    lam, V = np.linalg.eigh(H)
    U = V @ np.diag(np.exp(-1j * dt * lam)) @ V.conj().T

    rng = np.random.default_rng(0)
    psi = rng.normal(size=N) + 1j * rng.normal(size=N)
    psi /= np.sqrt(norm2(psi))

    n0 = norm2(psi)
    for _ in range(steps):
        psi = U @ psi
    nT = norm2(psi)

    print("norm(initial)  =", n0)
    print("norm(final)    =", nT)
    print("relative drift =", (nT - n0) / n0)


if __name__ == "__main__":
    main()
```

A.2 How to run (quick guide)

- Install dependency:

```
pip install numpy
```

- Run (default):

```
python3 jssh_sch_sanity_min.py
```

A.3 Interpretation

The relative norm drift should remain close to machine precision, typically in the range $\sim 10^{-13}$–10^{-15} depending on $(N, \Delta t, \text{steps})$ and floating-point rounding, confirming structural norm conservation. This numerical check is not a proof; it merely illustrates consistency with the analytical derivation presented in the main text.

A.4 Why this avoids circularity

This appendix does not assume the Schrödinger equation. It implements only the discrete conservative update

$$\psi^{(n+1)} = U\psi^{(n)}, \qquad U = \exp(-\mathrm{i}\Delta t\, H_d),$$

where H_d is constructed from local JS–SH adjacency (a discrete Laplacian limit). The observed norm preservation and the small-k dispersion are consequences of unitarity and locality in the discrete generator, not inserted continuum axioms.

Conflict of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Data Availability

The data and Python code supporting the findings of this study are available within the article and its supplementary materials.

SRCD–JS–SH Master Wave–Rotation Equation (Phase–Closure–Aware Unified Form)

Primitive Structural Variable (Absolute Axiom)

$$u := \frac{E}{r}, \qquad \text{all datasets and all interactions are represented only by } u. \tag{94}$$

Reference Ratio and Elastic Compactification

$$y(u) := \frac{u}{u_{\mathrm{ref}}}, \qquad \sigma(u) := \frac{y(u)}{1+y(u)}, \qquad 0 < \sigma(u) < 1. \tag{95}$$

Here $\sigma(u)$ is the *elastic ratio coordinate* used throughout SRCD; no centering, offset, or force-dependent tuning is permitted.

Elastic Phase Representation

$$\theta(u) := 2\pi\,\sigma(u), \tag{96}$$

where θ represents the accumulated structural phase induced by the ratio state $\sigma(u)$, with no arbitrary phase shifts or rotations.

Discrete Structural State (JS–SH)

$$\Psi_n(t) \in \mathbb{R}^2, \qquad J := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad J^{\mathsf{T}} = -J, \qquad J^2 = -I. \tag{97}$$

Each Ψ_n is a *two-channel real structural state.* The matrix J is the unique generator of orientation-preserving, norm-preserving internal rotation in $\mathbb{R}^2$. No complex numbers are assumed at the structural level.

Discrete Structural Master Equation (Above Schrödinger)

$$\boxed{\frac{d}{dt}\Psi_n = J\big[\Omega(\sigma(u_n))\,\Psi_n + \gamma_{\mathrm{SH}}(\Psi_{n+1} - 2\Psi_n + \Psi_{n-1})\big]} \tag{98}$$

Here:

- $\Omega(\sigma) := \Omega_0\,\sigma(u)$ is the *local structural phase-advance rate* induced by the elastic ratio coordinate, with $\Omega_0 > 0$ a fixed structural scale (no data fitting).
- $\gamma_{\mathrm{SH}} > 0$ is the symmetric SH-hub coupling strength, encoding nearest-neighbor structural connectivity.
- The second-difference term is the unique locality-preserving, translation-invariant SH transport operator.

This equation is the *fundamental dynamical law* of the framework. It is strictly norm-preserving and fully discrete. No quantum-mechanical postulates are used.

Continuum-Induced Master Law (Wave–Rotation Form)

$$\boxed{\partial_t\Psi(x,t) = J\Big[\Omega(\sigma(u(x)))\,\Psi + D\,\partial_x^2\Psi + D_4\,\partial_x^4\Psi + \cdots\Big]} \tag{99}$$

The coefficients $D, D_4, \ldots$ arise from the ordered-neighborhood expansion of SH adjacency. They are fixed by structural resolution and locality; no additional degrees of freedom are introduced. In the long-wavelength regime, the leading term is ∂_x^2.

Complex Representation (Structure-Hiding Compression)

$$\psi(x,t) := q(x,t) + i\,p(x,t), \qquad J\Psi \;\longleftrightarrow\; i\psi. \tag{100}$$

The complex field ψ is *only* a compact notation. Multiplication by i represents the action of J on the underlying real two-channel state.

$$\partial_t\psi = -i\Omega(\sigma(u))\psi + D\partial_x^2\psi \tag{101}$$

Energy Representation and Schrödinger Special-Case Limit

$$\hbar_{\text{eff}} := \text{structural frequency–energy conversion scale}, \qquad \frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}} := \hbar_{\text{eff}} D. \tag{102}$$

$$\boxed{i\hbar_{\text{eff}}\partial_t\psi = -\frac{\hbar_{\text{eff}}^2}{2m_{\text{eff}}}\,\partial_x^2\psi + V\,\psi} \tag{103}$$

This is the Schrödinger equation. In the present framework it is *not taken as a primitive postulate*: rather, it appears as the lowest-order, phase-open, long-wavelength reduction of the SRCD–JS–SH master wave–rotation equation shown above.

Structural Interpretation (ordering of representations).

SRCD ratio structure

$\Longrightarrow$

JS–SH discrete wave–rotation dynamics

$\Longrightarrow$

Schrödinger equation

(special continuum representation)

The Schrödinger equation therefore appears as a *structure-hiding* special continuum representation within the present framework.

In the main text, ω_0 denotes the homogeneous (background) structural rotation rate. The function $\Omega(\sigma)$ used here represents its SRCD-generalized form, allowing elastic ratio dependence without introducing new degrees of freedom or tuning parameters.

Summary of symbols. J: real rotation generator on the internal two-channel space; γ_{SH}: SH-mediated structural coupling strength; $\sigma(u)$: elastic ratio coordinate compactifying the primitive variable $u = E/r$.